Parameterized Post-Friedmann Formalism for the Dark Scattering Model

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Abstract

Recent results from the Dark Energy Spectroscopic Instrument find a preference for dynamical dark energy. This motivates improving the accuracy of predictions for dynamical dark energy models, including those which can resolve current tensions in the data, such as the dark scattering model. We improve the parameterized post-Friedmann approach for this model, reducing the error from approximately 1.3% to only 0.1%. Additionally, we show that the commonly used scale-independent approximation may not be completely accurate and, when applying the best-fit values from DESI on the dark scattering model, we predict an enhancement of the power spectrum at late times, worsening the S_8 tension.

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Introduction

On large scales, our universe consists of three main components: baryons, dark matter, and dark energy. Dark energy, which represents $\simeq 70\%$ of the universe, is responsible for accelerating the universe's expansion. While we understand the impact of dark energy on the universe, its fundamental nature remains unknown. However, several explanations for dark energy have been proposed such as the dynamical dark energy w_0w_aCDM (Chevallier *et al.* 2001), modified gravity (Tsujikawa 2010), and interacting dark energy models (van der Westhuizen *et al.* 2024).

The widely accepted standard cosmological model, ΛCDM, struggles to explain recent data discrepancies in cosmology such as the S_8 tension, which refers to a difference between the direct measurements of the clustering of matter in the late universe and the value inferred from the Cosmic Microwave Background (CMB) probing the early universe (Perivolaropoulos *et al.* 2022). As a result, several models beyond the ΛCDM model have appeared, including the Dark Scattering model (Simpson 2010), which introduces a scattering between the dark matter particles and the dark energy fluid similar to the Thompson scattering between electrons and photons. With the appropriate interaction strength, this model provides a promising explanation for the lower value of the S_8 parameter at late times.

Furthermore, the latest results from the Dark Energy Spectroscopic Instrument (DESI) indicate that the equation of state parameter of dark energy w – defined as its energy density divided by its pressure – crosses the phantom divide, corresponding to a value of $w = -1$ (DESI Collaboration *et al.* 2024). This motivates improving the accuracy of predictions of dark energy models that cross $w = -1$, and the Parameterized Post-Friedmann (PPF) approximation is the most general way to do that, which applies to most models of dynamical dark energy (Fang *et al.* 2008). This paper aims to enhance the accuracy of the PPF approximation for the dark scattering model to ensure that it can be confidently tested with the most precise data, minimizing potential biases.

PPF Formalism for the Dark Scattering Model

The PPF formalism offers a way of studying extensions to the standard cosmological model. It achieves this by introducing parameters that quantify those deviations within a well-defined formalism (Baker

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et al. 2013). In the context of dark energy, the PPF formalism has been tested in cases where the fluid approximation works well and then applied to cases where it breaks down, specifically those involving the crossing of the phantom divide $w = -1$.

In the fluid approximation, dark energy is treated as a fluid, allowing the application of hydrodynamic equations to describe its behaviour (Lesgourgues 2013). In the dark scattering case, the Euler equations for both dark energy and dark matter are modified by a drag term $a\xi_{ds}\rho_c\Delta\theta$, due to the scattering of the dark matter particles of the dark energy fluid (Simpson 2010), where

$$\xi_{ds} = \frac{\sigma_D}{m_c} \tag{1}$$

is the dark scattering parameter. The two Euler equations represent the evolution equations for the velocity perturbations for both dark energy e and dark matter c.

$$\theta_e' = 2\mathcal{H}\theta_e + k^2\Psi + k^2\frac{\delta_e}{1+w} - a\xi_{ds}\rho_c\Delta\theta \tag{2}$$

$$\theta_c' = -\mathcal{H}\theta_c + k^2\Psi + (1+w)a\xi_{ds}\rho_e\Delta\theta \tag{3}$$

The prime in the above equations denotes the derivative with respect to conformal time, $\frac{d}{d\eta}$. Here, a is the scale factor, and

$$\mathcal{H} = \frac{a'}{a} \tag{4}$$

is the Hubble parameter. In the second term, k is the wave number, and Ψ denotes the perturbation to the Newtonian potential. In the third term, δ_e represents the perturbation of the dark energy density. In the last term $\Delta\theta$ is the difference between the velocity perturbation of dark energy and dark matter, defined as $\Delta\theta = \theta_e - \theta_c$, where $\theta_e = \nabla \cdot V_e$ and $\theta_c = \nabla \cdot V_c$. In equation 2, the third term is undefined in the case of $w = -1$, which indicates the need for the PPF formalism. The construction of the PPF formalism replaces the fluid equations of dark energy with an alternative set describing the evolution of its fluctuations over the phantom divide. We follow the PPF approach as discussed in both Fang *et al.* (2008) and Li *et al.* (2014), and extend it to the dark scattering case.

Generally, in the PPF approach, a dynamical parameter Γ is introduced at large scales that reduces to the Poisson equation at small scales, where dark energy is assumed to be smoothed:

$$\Phi + \Gamma = \frac{4\pi G}{k_H^2 H^2}\Delta_T\rho_T \tag{5}$$

Here, Φ represents the perturbation to the spatial curvature and is related to Ψ in Equations 2 and 3 by $\Phi = -\Psi$ in the absence of anisotropic stress. This equation, excluding Γ, represents one of Einstein's equations in Newtonian gauge, as detailed in Hu *et al.* (1999) and Fang *et al.* (2008). On the right-hand side, $k_H = \frac{k}{aH}$ is the modified wave number, G is the gravitational constant, ρ_T is the total matter energy density, Δ_T is the density perturbation of matter – excluding dark energy – in the total matter gauge.

The equation of motion for equation 5, at all scales, is expressed as

$$(1 + c_\Gamma^2 k_H^2)[\Gamma' + \Gamma + c_\Gamma^2 k_H^2 \Gamma] = S \tag{6}$$

The introduction of c_Γ terms above imposes the physical condition that dark energy fluctuations vanish on sufficiently small scales ($k_H \gg 1$) as a consequence of having a sound speed close to that of light in most models of dark energy (which is also assumed here). As before, the prime denotes differentiation with respect to $\ln a$. The source term is represented by S.

In the case of generic interacting models, energy and momentum exchange contribute to the source term, S. However, only momentum exchange is considered for dark scattering, the contribution of which is denoted by f_c, as defined in Li *et al.* 2014. To find the correct expression of f_c in the case of dark scattering, we use the modified Euler equations (Eqs. 2 and 3), and find

$$f_c = \frac{\xi_{ds}}{k}\rho_c\Delta\theta(\rho_e + p_e) \tag{7}$$

We consider this expression when we derive the source term for the dark energy perturbations, S, and the resulting source term becomes

$$S = \left[\frac{4\pi G}{k_H^2 H^2}(\rho_e + p_e)\left(k_H V_T + \frac{3Z}{k_H}(V_T - V_c)\right) - \frac{3aZ}{F}c_\Gamma^2 \Gamma\right]\left[1 + \frac{3ZC}{k_H^2 F}\right]^{-1} \tag{8}$$

with

$$Z = \frac{\xi_{ds}\rho_c}{H}, \qquad C = 1 - \frac{1}{1 + c_\Gamma^2 k_H^2}, \qquad F = 1 + 3\frac{4\pi G}{k_H^2 H^2}(\rho_T + p_T) \tag{9}$$

Here, ρ_c and ρ_e are the dark matter and dark energy density, respectively. Both p_e and p_T are the dark energy and total matter pressure, respectively. The total matter velocity is represented by V_T. This result comes from deriving equation 5 and using both Equations 2 and 3. This formula represents our first main result, which enables us to improve the accuracy of predictions relative to the standard PPF formula (with $Z = 0$), demonstrated below.

Results

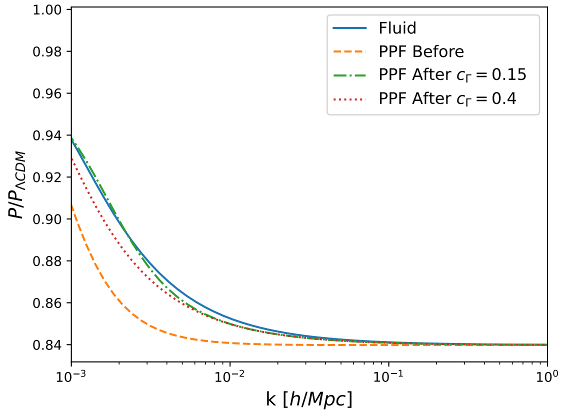
We implemented the new modified source term in the Cosmic Linear Anisotropy Solving System (CLASS) (Blas *et al.* 2011). We obtained an improved accuracy of predictions for the expected matter power spectrum ratio $P/P_{\Lambda CDM}$ from approximately 1.3% to 0.3% difference overall between the fluid result and the PPF result. A value of $c_\Gamma = 0.4$ is typically used for standard dark energy models, but this requires validation in the dark scattering case. We tested different c_Γ and found a value of $c_\Gamma = 0.15$ maximizes the accuracy of predictions, to approximately 0.1% difference overall. Fig 1a shows the resulting power spectrum ratio plotted against the scale, k, before and after modifying the source term in the PPF approximation, compared to the fluid approximation in a case with constant w. This improved accuracy allowed us to test different scenarios of the dark scattering model crossing the phantom divide, where we consider the Chevallier-Polarski-Linder (CPL) parameterization for the equation of state

$$w(a) = w_0 + w_a(1 - a) \tag{10}$$

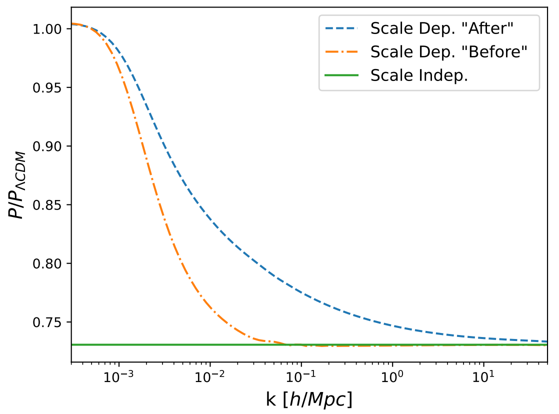
as in Chevallier *et al.* (2001) and Linder (2003).

We show an interesting case in Figure 1b, where the power spectrum ratio is plotted before and after correction for the case of $w_0 = -1.15$ and $w_a = 0.5$. The difference between the two is notable, demonstrating the improved accuracy our predictions following the above modifications. Figure 1b also indicates that the commonly used scale-independent approximation for the power spectrum in the range of $10^{-2} < k < 10^1$ may not be accurate in some cases. This approximation is generally valid on small scales (i.e., large values of k) because the dark matter equation does not depend on the scale and dark energy fluctuations are considered negligible. With the help of the scale-independent approximation presented in Carrilho *et al.* (2022) for the dark scattering case, we show the difference between the scale-independent approximation and the full calculation labelled "After" in Figure 1b. The difference between the two indicates the need to use the modified source term in analysing real data when considering dark scattering.

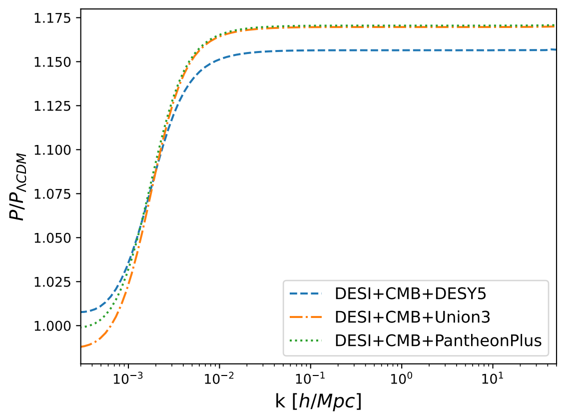
Applying the best-fit values for w_0 and w_a from DESI Collaboration *et al.* (2024), we find that the dark scattering model does not suppress the power spectrum, but rather enhances it at late times. Figure 1c shows the resulting power spectrum ratio from the best-fit values from the three SN Ia data sets. This result indicates that, if the universe is described by these values of w_0 and w_a, then the dark scattering model is not a solution for the S_8 tension, which requires instead a suppression at late times.



(a)



(b)



(c)

Figure 1: The power spectrum ratio $P/P_{\Lambda CDM}$ is plotted against the scale k at $z = 0$ using a value of $\xi_{ds} = 50\text{b/GeV}$. (a) The PPF approximation before and after modifying the source term is compared with the fluid approximation for the dark scattering case $w = -0.9$. (b) The plot shows the crossing of the $w = -1$ case with the following values $w_0 = -1.15$ and $w_a = 0.5$ before and after modifying the source term, compared with the scale-independent approximation. (c) The three DESI cases correspond to the central values from the three SN Ia data sets: $w_0 = -0.727$ and $w_a = -1.05$, $w_0 = -0.64$ and $w_a = -1.27$, $w_0 = -0.827$ and $w_a = -0.75$.

Conclusions

In this paper, we improve the accuracy of the PPF formalism for the dark scattering model, enabling confident testing of phantom crossing cases for this model. A notable result revealed by our predictions following modifications, is the substantial scale dependence of the power spectrum ratio within the scale range $10^{-2} < k < 10^{1}$ in some cases, requiring careful analysis of real data within this scale range. We also make predictions for the values of w_0 and w_a favoured by DESI, finding that if they accurately represent the real universe, the dark scattering model cannot resolve the S_8 tension, as it would instead enhance clustering. A detailed exploration of this model with the DESI data is therefore needed in the future.

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References

Baker, T. *et al.* 'The Parameterized Post-Friedmann Framework for Theories of Modified Gravity: Concepts, Formalism, and Examples' Physical Review D **87** 2 (2013)

Blas, D. *et al.* 'The Cosmic Linear Anisotropy Solving System (CLASS). Part II: Approximation Schemes' Journal of Cosmology and Astroparticle Physics **2011** 07 (2011)

Carrilho, P. *et al.* 'On the Road to per Cent Accuracy VI: The Non-linear Power Spectrum for Interacting Dark Energy With Baryonic Feedback and Massive Neutrinos' Monthly Notices of the Royal Astronomical Society **512** 3 (2022)

Chevallier, M. and Polarski, D. 'Accelerating Universes With Scaling Dark Matter' International Journal of Modern Physics D **10** 02 (2001)

DESI Collaboration *et al.* 'DESI 2024 VI: Cosmological Constraints From the Measurements of Baryon Acoustic Oscillations' arXiv e-prints (2024)

Fang, W. *et al.* 'Crossing the Phantom Divide With Parametrized Post-Friedmann Dark Energy' Physical Review D **78** 8 (2008)

Hu, W. and Eisenstein, D. J. 'Structure of Structure Formation Theories' Physical Review D **59** 8 (1999)

Lesgourgues, J. 'TASI Lectures on Cosmological Perturbations' arXiv e-prints (2013)

Li, Y.-H. *et al.* 'Parametrized Post-Friedmann Framework for Interacting Dark Energy' Physical Review D **90** 6 (2014)

Linder, E. V. 'Exploring the Expansion History of the Universe' Physical Review Letters **90** 9 (2003)

Perivolaropoulos, L. and Skara, F. 'Challenges for ΛCDM: An Update' New Astronomy Reviews **95** (2022)

Simpson, F. 'Scattering of Dark Matter and Dark Energy' Physical Review D **82** 8 (2010)

Tsujikawa, S. 'Modified Gravity Models of Dark Energy' in Lectures on Cosmology (Springer Berlin Heidelberg; 2010)

Van der Westhuizen, M. A. and Abebe, A. 'Interacting Dark Energy: Clarifying the Cosmological Implications and Viability Conditions' Journal of Cosmology and Astroparticle Physics **2024** 01 (2024)