A Topological Investigation of the Cosmic Web Formation

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Abstract

N-body simulations of the evolution of the large-scale structure of the universe (the Cosmic Web) were run while altering attractive gravity laws and initial conditions, in order to infer which properties of the universe are revealed by its current large-scale topology. The Cosmic Web was found to develop regardless of the gravitational alterations made. Significantly altering the initial conditions from Gaussian distributions was found to eliminate the Cosmic Web. Thus, we determine that small Gaussian-like perturbations are required in otherwise uniform initial conditions in the early universe, under an attractive gravitational force, for the Cosmic Web to develop.

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Introduction

The large-scale structure of the universe, known as the Cosmic Web (CW) (Bond *et al.* 1996), is characterised by: knots, collapsed regions that host galaxies and halos; sheets and filaments, regions collapsed in one or two dimensions respectively that connect the knots; and voids, under-dense regions in between (Martizzi *et al.* 2019).

Modelling matter as collisionless fluid described by continuous phase space density $f(\mathbf{x}, \mathbf{v})$, a function of position, $\mathbf{x}$, and velocity, $\mathbf{v}$, the evolution of the CW can be described by the Vlasov-Poisson system of equations (see e.g., Rein 2007):

$$\frac{\partial f}{\partial t} + \nabla_{\mathbf{x}} f \cdot \frac{\mathbf{v}}{a^2} - \nabla_{\mathbf{v}} f \cdot \frac{\nabla_{\mathbf{x}} \phi}{a} = 0 \tag{1}$$

$$\rho(\mathbf{x}, t) = m_{\mathbf{x}} \int f(\mathbf{x}, \mathbf{v}, t) \mathrm{d}^3 v \tag{2}$$

$$\nabla_{\mathbf{x}}^2 \phi = -4\pi G \rho_0 \delta \tag{3}$$

where a is the cosmic scale factor, ϕ is the gravitational potential, ρ is the local density, ρ_0 is the average density, and δ is the fractional difference between ρ and ρ_0. Equation 3 is known as the Poisson equation (Poisson 1827). This system of equations cannot be analytically solved, thus simulations are needed to solve the equations numerically.

This work uses N-body simulations to study the evolution of the CW while varying the laws of gravity and initial universe conditions, in order to infer which properties of the universe are revealed by the current large-scale topology of the universe. This is done by trying to 'break' the CW by making two classes of modifications to the simulations: modifying the Poisson equation, and altering the initial conditions of the universe. 2D simulations were used since they require significantly less computing power than 3D simulations, as to maximise the amount of alterations tested.

The code for the 2D N-body simulations used was based on code written by Stücker (2019). The initial form of the simulation started with a uniform grid of particles, which were then given small perturbations to their positions as initial conditions, randomly selected from a Gaussian distribution. The Vlasov-Poisson system of equations was then solved numerically, along with equations 4 and 5 which

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describe the evolution of the particle position and momentum vectors, X^i and P^i, for a particle i in the weak-field non-relativistic limit (Angulo *et al.* 2022), to calculate the evolution of the potential, ϕ, with fractional density changes, δ. Thus, the evolution of the position of each particle was determined based on the evolution of ϕ:

$$\frac{\mathrm{d}X^i}{\mathrm{d}t} = \frac{P^i}{m} = \frac{P_i}{ma^2} \tag{4}$$

$$\frac{\mathrm{d}P_i}{\mathrm{d}t} = -m\frac{\partial \phi}{\partial X^i} \tag{5}$$

Poisson Equation Modifications

The first method used to attempt to prevent the CW from forming in the simulations was to make modifications to the Poisson equation. Alterations made to this equation changed the strength of gravity in different ways, while keeping the force attractive. The two main modifications to the Poisson equation that were tested are called the Different Derivative Model (DDM) and the Different Response Model (DRM).

The DDM tested changing the strength of gravity by changing the order of the derivative in the Poisson equation, thus varying the value of n in:

$$\nabla^n_{\mathbf{x}}\phi = -4\pi G\rho_0\delta \cdot R^{2-n} \tag{6}$$

where R is a constant used to ensure the units are the same on both sides of the equation. Increasing n results in an accelerated and stronger gravitational response to density changes, while decreasing n results in a decelerated and weaker gravitational response.

The DRM instead changes the strength of gravity by changing the dependence of the field on δ in the Poisson Equation, thus varying the value of n in:

$$\nabla^2_{\mathbf{x}}\phi = -4\pi G\rho_0\delta^n \tag{7}$$

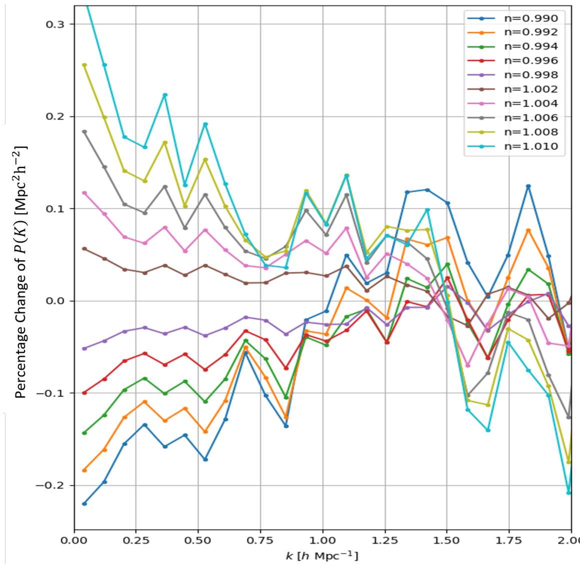
where again, larger values of n result in stronger gravitational responses to density changes.

Running simulations using both models for various values of n from $a = 0.02$ to $a = 1$ and beyond, a clear pattern emerges. Starting from the Gaussian initial conditions, a structure with the same topology as the CW would develop, which would eventually be destroyed as the knots would merge into larger collapsed regions which would start absorbing the filaments. The main difference between these simulations was the time at which the CW developed, and when it was subsequently destroyed. While the CW in our initial simulation is fully formed at $a = 1$, increasing n caused it to develop earlier, and decreasing n caused it to develop at higher values of a. For example, using $n = 1.25$ in a DRM simulation caused the CW to have fully developed by $a = 0.15$, which can be seen in Figure 1b. Based on these results, we can conclude that changing the strength of gravity cannot prevent the CW from eventually forming. This also shows that the topology of the CW alone cannot tell us which of these models represents our universe, and thus other cosmological observations are needed to put constrains on which models are possible, such as the age of the oldest stars in the universe.

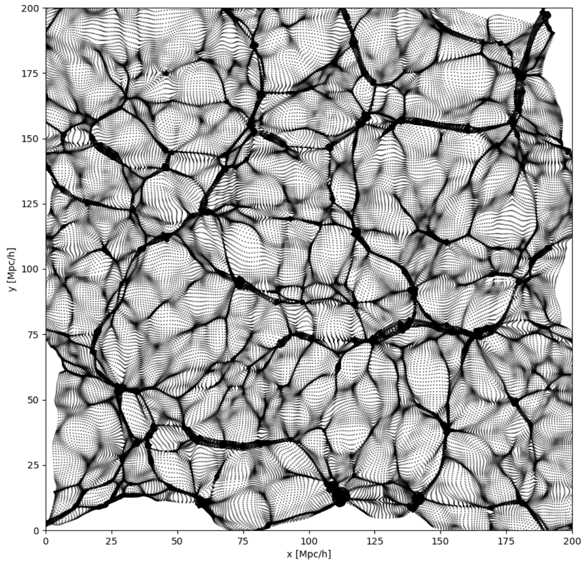
The differences between these models were quantified through the calculation and comparison of their power spectra and their probability distribution function statistics. The comparison of the power spectra for different values of n to the original value of n in both models showed a different absolute fractional change between positive and negative changes to n of the same magnitude. An example of this for the DRM can be seen in Figure 1a.

Initial Conditions Modifications

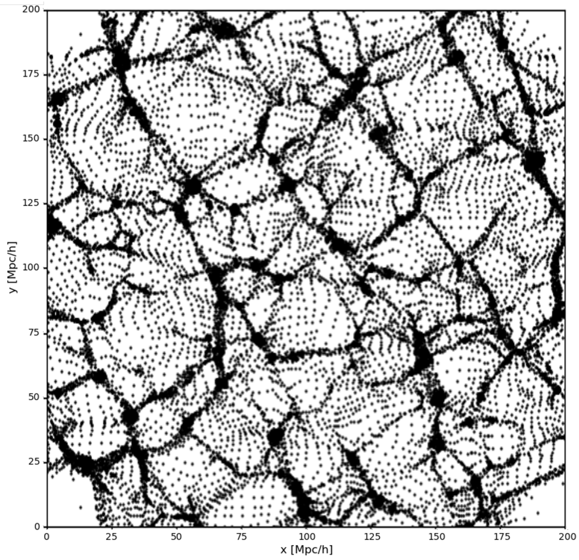
The second method used to attempt to prevent the CW from forming in the simulations was to change the initial conditions provided to the original simulation, which solves the unaltered Vlasov-Poisson system of equations. Instead of randomly selecting small perturbations from a Gaussian distribution as initial conditions for each particle, different distributions were used to select the initial perturbations of the particles.



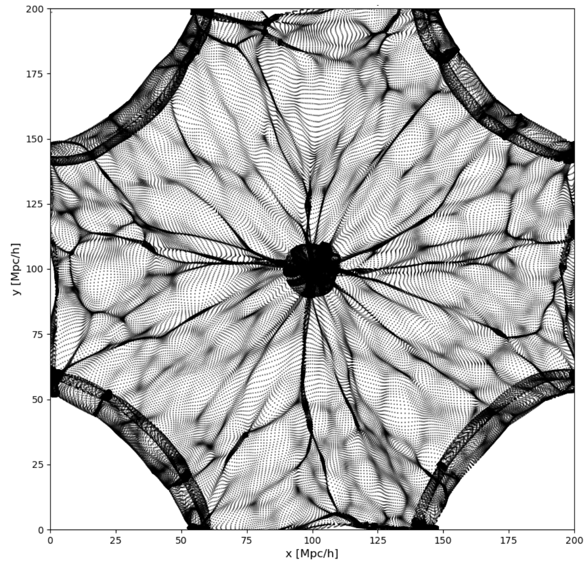
(a) The fractional change of the DRM $n = 1$ power spectrum when changing the value of n at different scales in Fourier space.



(c) Simulation results at $a = 1$, using the unaltered Vlasov-Poisson system of equations and a Poisson distribution as initial conditions.



(b) Simulation results at $a = 0.15$ using the DRM with $n = 1.25$, using a Gaussian distribution as initial conditions.



(d) Simulation results at $a = 1$, using the unaltered Vlasov-Poisson system of equations and a Binomial distribution ($n = 10$, $p = 0.9$) as initial conditions.

Figure 1: Multiple plots showing simulation results and statistics for three of the alterations made to the simulation.

Using a Poisson distribution and a Random/Uniform distribution to select particle initial conditions had the same result: a structure with the same topology as the CW having developed at $a = 1$, which only had small density differences with Gaussian distribution initial condition results. Using a Binomial distribution with a high p value to select initial conditions, which has a very small overlap with a Gaussian distribution, resulted in the CW not forming in the simulation at any value of a. Two examples of the simulation results using different initial condition distributions can be seen in Figure 1c and 1d.

The last initial condition modification used in order to take a further step away from the Gaussian distribution was using images to define the initial conditions. This was done by firstly converting the images to gray-scale, which assigns each pixel a number describing where it is in the black-white spectrum. Each particle was then assigned the pixel corresponding to its location index, and the magnitude of its perturbation was the gray-scale number of the pixel. This resulted in the grid of particles at $a = 0.02$ looking similar to a less resolved or pixelated version of the image. Using this method of defining particle initial conditions also prevented the CW from developing at any point in the simulation.

These results leads us to the conclusion that small perturbations, following a distribution similar to a Gaussian, to otherwise uniform initial conditions are required for the CW to form. Thus, the current topology of the CW reveals information about the initial conditions of the universe, billions of years ago.

Conclusions

From the altered simulations run in this project, we can conclude that the strength of gravity does not affect the topology of the Cosmic Web. This large-scale structure develops regardless of the type or strength of gravitational alterations made. The formation of the Cosmic Web can be prevented by altering the initial conditions given to the simulation to ones significantly different from a Gaussian distribution, such as a Binomial distribution with a high p value. In conclusion, small Gaussian-like perturbations to otherwise uniform initial conditions of particles (under the influence of an attractive gravitational force) are required in order to generate the Cosmic Web. This tells us that the current topology of the Cosmic Web carries information about the early universe's conditions.

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